[5] Tribus, M., "Comment on 'Fundamentals of boundary-layer heat transfer with streamwise temperature variations,'" J. Aerospace Sci. 29, 1482–1483 (1962).

Reply by Author to M. W. Rubesin

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THE full text of assumption 5 in the conclusions of Rubesin's paper (Ref. 2 of the preceding comment) reads as follows: "The local heat-transfer coefficient determined on a plate having a constant surface temperature applies to a plate having a variable surface temperature when it is expressed by an equation based on the local flow and thermal boundary-layer thicknesses instead of the distance along the plate."

In the foregoing context, the concept of local heat-transfer coefficient embodies a physical model, which is fundamentally wrong. In addition to being inadequate to cope with the anomalies, it also misrepresents the real physical nature of the phenomenon. This is shown clearly and explained in my analysis of what happens inside the fluid by introducing a two-dimensional model.

In two recent papers (Refs. 1 and 4 of the preceding comment), I have developed a new approach to transient temperature analysis in structures in contact with a moving fluid in laminar or turbulent flow. The significance of this work is being obscured by comments that are focused on a side issue.

In order to restore true perspective and dispel some of the misconceptions evidenced by the comments, I should like to state more precisely what has been accomplished.

1) A correct formulation of transient temperature problems in a system that includes coupling between conduction in a solid and convection in an adjacent moving fluid leads to partial differential equations with integro-differential boundary conditions. They are impractical for use in structural analysis. However, the new Lagrangian variational procedure (Ref. 4 of the preceding comment) avoids this difficulty while retaining a correct representation of the physics. The problem is reduced to the solution of a system of ordinary differential equations with a few generalized coordinates. The treatment also brings to light some new and fundamental aspects of nonequilibrium thermodynamics for which Onsager's relations do not apply.

2) Additional simplifications of practical importance are provided by extending to convective phenomena the concept of "associated field." It is shown to be applicable in spite of the fact that the problem is not self-adjoint.

3) As a corollary, I have developed in Ref. 1 of the preceding comment a new approach, also based on variational procedures, for the conventional more restricted problem of boundary-layer heat transfer, including turbulence and nonparallel streamlines in two and three dimensions. The method combines simplicity with high accuracy and brings to light the significant parameters.

4) Without the use of new methods, a correct formulation of transient temperature problems with surface convection is so involved that it has been customary to represent the boundary condition by a local heat-transfer coefficient. The first portion of the first paper (Ref. 1 of the preceding comment) is devoted to an examination of the misconceptions involved in this procedure by analyzing very simple mathematical models that highlight the essential features and provide a deeper insight into the physics. It points to the key role played by the Peclet number as a measure of the distortion of the temperature field and its relation to a thermal flow reversal inside the fluid. This is in contrast with the customary use of the Reynolds and Prandtl numbers, which are not representative of the physics involved.

5) This physical analysis of convective phenomena is based on the classical procedure used by Leveque, which amounts to introducing a "conduction analogy." Since this in itself is an approximation, its validity and limitations were examined in the first paper (Ref. 1 of the preceding comment), with particular reference to transient phenomena.

6) It is shown that the unorthodox behavior of the convective heat transfer is inherent in the physics and does not result from spurious mathematical properties. Any attempt to camouflage it by mathematical juggling misrepresents its real nature.

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Comment on "Fundamentals of Boundary-Layer Heat Transfer with Streamwise Temperature Variations"

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THE author of Ref. 1 replies to a comment by Tribus[2] and maintains that the latter's statements "are in gross contradiction with the facts." I wish to confirm that these statements are not in contradiction with the facts.

Negative and infinite "traditional" heat transfer coefficient mentioned in Ref. 1 already has been predicted in the last reference cited in Ref. 2. The writer has treated[3,4] the problem of sinusoidal heat flux and obtained another extension of Leveque's solution. The "traditional" coefficient is defined with the remark that "it is of little use since there is no connection with other relevant quantities of heat transfer."

References

[1] Biot, M. A., "Fundamentals of boundary-layer heat transfer with streamwise temperature variations," J. Aerospace Sci. 29, 558–567, 582 (1962).

[2] Tribus, M., "Comment on 'Fundamentals of boundary-layer heat transfer with streamwise temperature variations,'" J. Aerospace Sci. 29, 1482–1483 (1962).

[3] Dzung, L. S., "Heat transfer in a round duct with sinusoidal heat flux distribution," *Proceedings, Second U.N. Conference on Atomic Energy* (United Nations, Geneva, 1958), Vol. 7, p. 657.

[4] Dzung, L. S., "Heat transfer in a flat duct with sinusoidal heat flux distribution," Ref. 3, p. 671.

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Reply by Author to L. S. Dzung

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IT should be obvious, after reading my foregoing reply to Rubesin's comments, that much more is involved here than a passing reference to unorthodox and often misinterpreted analytical properties. The real question is why does this happen? Is it more than a spurious mathematical property, and what is its more intimate physical nature? This I have

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